Draft Issue

The purpose of this paper is to elicit information on the method I have used over the past 2 years to run dynamic process simulation. The key elements of my simulation work has involved dynamic simulation of compressors, pipes, valves, and heat exchangers. In general, the software I've used is not ideally suitable to dynamic simulation of mixed phases, and there have been considerable progress made by the process simulation packages towards multicomponent fractionation towers. In the first instance my dynamic simulation was initiated to model the depressurization cycle of a dehydrator bed and evaluate control loop tuning for minimizing pressure swings. The key components to the dehydrator system were gas control valves, gas piping systems, and fixed volume beds. In the first instance the simulation was taken on as an intellectual challenge to determine whether or not the simulation could accurately model the time pressurization curves for the dehydrator beds for pressurization and depressurization times, largely to assist in making sizing calculations on the flow elements to optimize the depressurization and repressurization steps. The initial success, was encouraging enough to stimulate additional work on the flow control valve loops and control strategies to evaluate strategies to minimize pressure bumps resulting from swings in the gas flow paths. The evaluation was successful in that it identified that simple control tuning modifications would not substantially improve the pressure bumps which resulted from flow changes. The process & controls simulation indicated that the extensive pipe & valve changes to reduce the pressure bumps were not an economical alternative, in view of other alternatives under consideration.

The next opportunity to utilize dynamic simulation was to evaluate the effectiveness of pressure control strategies for BI-3128. The dynamic simulation determined that it would be necessary to reconfigure a pressure control for proper operation. This dynamic simulation used a process simulator for making determination of physical properties, while the dynamic simulator was used to evaluate the control valve and control elements. This success in making review of control strategy and confirm the proper control configuration was encouraging.

On the same project, a dynamic simulation was made of the compressor startup sequence. This simulation was able to confirm that it would be possible to minimize construction cost and to implement a heat exchanger bypass, to eliminate the potential for hydrate formation, without flaring during the startup. Again, the dynamic simulation used a process simulator for making determination of physical properties, while the dynamic simulation was used to evaluate the control valves and control elements, plus evaluate thermal transients and simulate compressor startup time and process and heat flows during compressor speed up.

Attached are the basic methodologies for implementing of the calculations of dynamic simulation. The paper presents a novel method for evaluating momentum transfer by use of the Lagrange method to field engineering problems. Typically, the classic engineering texts concerned themselves with avoiding dynamic calculations, in favor of eliminating the differential equations for kinematic analysis. With the advent of low cost and effective ODE solvers that can run on high end PC's, the dynamic simulation method is now available to field engineers at a reasonable cost.

Particularly unique to this paper is the extension of the Lagrange method to engineering problems for development of dynamic equations. The best example is the dynamic equation for gas flow inside a centrifugal impeller, sections 4 & 9. Also presented is an improved approach to dynamic heat transfer which accounts for the transients due to the container wall, section 17 & 18. These 2 improvements make this publication noteworthy as an extension of present engineering method, in my opinion.

BACKGROUND APPENDIX

1. INTRODUCTION

This method is generally applied to turbo machinery where torque's are more important than forces.(Streeter p167) Engineering equipment where these principles are applied are power generation turbines, centrifugal pumps, compressors, centrifuges, cyclones, modified centrifugal pumps used for particle & solids separation under centrifugal force (Ledbetter/Perry), rotating spray heads, rotating mass transfer units such as the HiGee unit and cyclone separation drums, and electric motors. In summary, a large percentage of common engineering equipment utilizes the principle of rotation for production of useful work. Rotational principles are applied from the basic wheel for transportation to orbital satellites used in communications systems. The analysis concepts for these items are generally presented on an as-needed basis. The purpose of this discussion is to present a collection of the methods in comprehensive document The goal is to present the methods available for dynamic simulations and analysis. Karnopp (p351) states that analysis methods may be lumped into 2 broad categories, Euler and Lagrange. The Lagrange methods being generally applied to systems without mass flux. The Euler method being applied to systems where a control surface & volume are significant. Where possible, this paper will tend towards development of differential equations that can be numerically integrated. Performing anything beyond basic control surface integration is to be avoided since numerical integration is easily handled by simulation routines. Where-as analytical integration cannot be relegated to computational codes. In any system of analysis, a coordinate system must be selected. The coordinate system is the environment which best describes the system. There are various examples of coordinates systems such as x, y, z, t; or r, θ, t. Coordinates may be mixed as required to define the problem, such as temperature, pressure, time and distance. Selection of coordinate system is to provide the desired information with the minimum number of variables and simple equations. Having once selected a coordinate system, the Hamilton variation of the Lagrange method states:

$d(\partial \mathbf{L}/\partial Vs)/dt - \partial \mathbf{L}/\partial Xs = 0$ for (s= 1, 2, 3, n) (Nussbaum Groups p102, Boas p384)

$\mathbf{L} = \mathbf{T} - \mathbf{U} \equiv$ Lagrangian
V is coordinate velocity, dXs/dt best considered as a variable
X is coordinate
T is kinetic energy
U is potential energy (NB: $\partial \mathbf{U}/\partial X_s = \mathbf{F}_s$

Example, for 3 balls connected with 2 springs, center sharing common spring to the ends.

$Us = \int Fdx = \int kXdX = ½KX^2$, there are 3 coordinate, 1 for each ball

$\mathbf{U} = ½(k_a[X_1 - X_2]^2 + k_b[X_2 - X_3]^2)$

$T = ½(m_1[V_1]^2)$ & $\mathbf{T} = ½(m_1[V_1]^2 + m_2[V_2]^2 + m_3[V_3]^2)$

$\mathbf{L} = \mathbf{T}\text{-}\mathbf{U} = ½(m_1[V_1]^2 + m_2[V_2]^2 + m_3[V_3]^2) - ½(k_a[X_1 - X_2]^2 + k_b[X_2 - X_3]^2)$

$\partial \mathbf{L}/\partial X_1 = k_a[X_1 - X_2]$ & $\partial \mathbf{L}/\partial X_2 = k_b[X_2 - X_3] - k_a[X_1 - X_2]$ & $\partial \mathbf{L}/\partial X_3 = -k_a[X_2 - X_3]$

$\partial \mathbf{L}/\partial V_1 = m_1[V_1]$ & $\partial \mathbf{L}/\partial V_2 = m_2[V_2]$ & $\partial \mathbf{L}/\partial V_3 = m_3[V_3]$

$d(\partial \mathbf{L}/\partial V_1)dt = m_1[dV_1]/dt$ & $d(\partial \mathbf{L}/\partial V_2)/dt = m_2[dV_2]/dt$ & $d(\partial \mathbf{L}/\partial V_3)/dt = m_3[dV_3]/dt$

Finally the three coordinate equations of motion are:

$m_1[dV_1]/dt + k_a[X_1 - X_2] = 0 = m_2[dV_2]/dt + k_b[X_2 - X_3] - k_a[X_1 - X_2] = m_3[dV_3]/dt + -k_a[X_2 - X_3]$

The three auxiliary equations are $[dV_n]/dt = d^2X/dt^2$. The auxiliary equations may be direct substituted on solved simultaneously, depending on initial conditions. This easily accomplished with a simulator package such as Vissim.

2. METHOD OF LAGRANGE TO FLUID FLOW:

Considering two dimensional flow, define **T** in terms of average velocity, $V_1^2/(2g)$ & $\mathbf{U} = p/\rho + X\sin\theta$

The term $X\sin\theta$ is the height above the datum, or elevation head.

$\mathbf{L} = \mathbf{T} - \mathbf{U} = V_1^2/(2g) -(p/\rho + X\sin\theta)$

$\partial \mathbf{L}/\partial X_1 = -\partial(p/\rho + X\sin\theta)/\partial X_1 = -[\partial(p/\rho)/\partial X_1 + (\sin\theta)]$, at constant density

$\partial \mathbf{L}/\partial V_1 = \partial(V_1^2/(2g))/\partial V_1 = V_1/(g)$

$d(\partial \mathbf{L}/\partial V_1)dt = d(V_1/(g))dt = (1/g)\, dV_1/dt$ Finally the coordinate equations of motion are:

$dV_1/dt + g\{\partial(p/\rho)/\partial X_1 + (\sin\theta)\} = 0$

$dV_1 = dX_1[\partial V_1/\partial X_1] + dt[\partial V_1/\partial t]$ and the total derivative is

$dV_1/dt = [dX_1/dt]\,\partial V_1/\partial X_1 + dt/dt\,\partial V_1/\partial t = V\partial V_1/\partial X_1 + \partial V_1/\partial t \cong \Delta V_1^2/(2dX_1) + \partial V_1/\partial t$

The transform is made by $(dX_1/dt) = V$ & $VdV = \Delta V^2/2$ & finally

$\partial(V_1)/\partial t + \Delta V_1^2/(2\, dX_1) + g\{\partial(p/\rho)/\partial X_1 + (\sin\theta)\} = 0$ or with friction losses

$\partial(V_1)/\partial t + \Delta V_1^2/(2\, dX_1) + g\{\partial(p/\rho)/\partial X_1 + (\sin\theta)\} + f|V|V/2D= 0$ or I

The above reduces to Bernoulli equation for steady state velocity $[\partial V_1/\partial t =0]$ as:
$\Delta V_1^2/(2g) + \{\Delta(p/\rho) + \Delta X_1(\sin\theta)\} = 0$ Which confirms the method for flowing systems. When solving water hammer, the continuity equation provides the necessary information to solve for a velocity profile, $V(x,t) = Q/A$, given without derivation, for constant density, from Streeter & Pump Hdbk

$1/\rho\, dp/dt + a^2\partial V/\partial X = 0$ a (fps) $\equiv [gK/\rho/(1+K/Y(D/t')C_1)]^{1/2}$ C_1 typically being = 1.
The bulk modulus of elasticity is the media density times the square of sonic velocity for the fluid material divided by g=32.2 ft/s/s. Typical values of a are between 3000 & 5000 fps.

CONDITION	C_1, $\mu \equiv$ Poisson's ratio = .3
1 end fixed	1.25-μ
both ends fixed	1-μ^2
both ends fixed + expan joint	1-μ/2

MATERIAL	BULK MODULUS	SG 1=62.4PCF	MATERIAL	YOUNGS MODULUS
WATER 60F	45 mm psf	0.9995	steel	4032 mm psf
WATER 212F	39	0.956	cast iron	2304
CRUDE OIL	24-30	0.7-0.9	brass	1872
NGL	13	0.45-0.65	copper	2016
BENZENE	21	0.88	glass	1440
MERCURY	56	13.6	transite/rubber	490/288

When dealing with compressible fluid, the above can be transformed to:

$$\partial G/\partial t = -144g/L\{\Delta(psi) - \Delta G^2/(2\rho g*144) - \Delta p_f\} \quad 1$$

$$\partial \rho/\partial t = \{G_2(t) - G_1\}/L \quad 2$$

In all cases, both the initial values and steady state values of the terms must be checked to ensure physical reality prevails. Additional details on compressible fluid dynamics are given below.

3. BASIC ANGULAR TERMS

The definitions of angular velocity and acceleration are; $d\theta/dt = \omega$ & $d\omega/dt = \alpha$ where θ is angle of rotation in radians, ω is angular velocity in radians/second or in RPM as N, & α is acceleration, radians per second per second or dN/dt in RPM per second. Note that $\omega r = V_t$

For α = zero, $\theta = \theta_o + \omega * t$ t is time in seconds 1

For α = Constant, $\theta = \theta_o + \omega * t + ½\alpha t^2$ & $\omega = (\omega_o + \alpha t)$ & ${}^1/_2\Delta\omega^2 = \alpha\Delta\theta$ 2

For $\alpha \neq C$, then general forms are required.

For polar-cylinder coordinates $ds^2 = dr^2 + (rd\theta)^2 \Rightarrow v^2 = (ds/dt)^2 = (dr/dt)^2 + (r(d\theta/dt))^2$

Lets try the Lagrange equation for planetary motion. The kinetic energy, KE, in a centrifugal field of a particle is $½mv^2$, and potential energy by Newton's Universal gravity constant as GMm/r. The value of G in engineering units is approximately $3.44x10^{-8}$ $ft^4/lb/s^4$.

T $=½m\{(dr/dt)^2 + (r\omega)^2\}$ & define potential energy by Newton's gravity law as **U** = (GMm/r),

L = T - U (let V_i stand for the time derivative of the coordinate variable)

$\partial\mathbf{L}/\partial r = \partial(½m\{(dr/dt)^2 + (r\omega)^2\})/\partial r_1 - \partial(GMm/r)/\partial r_1 = m(r\omega)\omega + GMm/r^2$

$\partial\mathbf{L}/\partial V_r = ½m\partial\{(dr/dt)^2 + (r\omega)^2\}/\partial V_r = ½m\,\partial\{(V_r)^2 + (r\omega)^2\}/\partial V_r = m\,V_r$

$\partial \mathbf{L}/\partial V_\theta = ½m\partial\{(dr/dt)^2 + (r\omega)^2\}/\partial\omega = ½m2r\omega\partial\{(r\omega)\}/\partial\omega = mr\omega r = m\omega r^2$

$d(\partial \mathbf{L}/\partial V_r)dt = md(V_r)/dt = (m)\, dV_r/dt$

$d(\partial \mathbf{L}/\partial V_\theta)dt = (m)d(\omega r^2)/dt = (m)\, [d\omega r^2/dt = \omega dr^2/dt + r^2 d\omega/dt] = (m)[2r\omega V_r + r^2\alpha]$

Finally the coordinate equations of motion based on $d(\partial \mathbf{L}/\partial Vn)/dt - \partial \mathbf{L}/\partial n = 0$ are:
for r:

$mdV_r/dt - \{m\,(r\omega)\omega + GMm/r^2\} = 0 \Rightarrow dV_r/dt - (r\omega)\omega = -GM/r^2$

for θ:
$m[2r\omega V_r + r^2\alpha] = 0 = r\omega V_r + r^2\alpha = 2\omega V_r + r\alpha = 0$

These equations for planetary motion can be solved in terms of astronomical units of 1 sidereal year and earth/sun distance units, along with elliptical properties and kepler's laws to obtain initial values. Kepler states $a^3/p^2 = GM_T/4\pi^2$, from which $G = 4\pi^2$ (Intro. to Astrophysics, Dover Pubs) The other *initial* values are obtained from the following elliptical properties:

$B=a(1-e^2)$, $r = B/(1-e^*\cos\theta)$, $\beta=[\pi a^2\sqrt{(1-e^2)}]$; $dV_r/dt = -[4\beta^2\, e^*\cos\theta]/r^2B$ & $\omega=2\beta/r^2$

Where e is the eccentricity of orbit, p is orbital period, in earth years, a=1 for earth & others by Kepler's law. Fraleigh, Vol 2, section 10 & 3.1. Below is a table of results. The average error in the closure time about one percent, with most being far better. The largest errors were in calculation of mercury and Pluto closure times. The orbital calculations could be improved by using 2 important secondary considerations. The biggest improvements come from formulating the problem with consideration of effects from all mass points, multi-body problem, as with the triple coordinate springs. A second correction is the inclusion Einstein's correction to centrifugal force in a mass field. The relative coordinate system

$ds^2 = (1-A/r)dl^2 - dr^2/(1-A/r) - (rd\theta)^2$ & $(1-A/r)^2(dl/ds)=\text{constant}$, & $A= 2GM/c^2$

This correction factor is to advance the position of the semi major axis at a rate of $24\pi^3(a/Tc)^2$ radians per revolution. Where a is semi-major axis length, T is time for one revolution, and c is speed of light in-vacuum. However this correction is only about 1 revolution of the semi-major axis per 1 million years, 43 seconds of arc per year for mercury. The gravity force correction causes rotation to advance. The effect of round-off error is seen in the result for earth orbit. The calculation of Mercury orbit is known to be difficult due to competing gravity effects of other planets, the error of Pluto is from poor data. There was a variation of, almost 1 year between 2 sources for the periods of revolution for Pluto, and some 5 million miles difference between the estimates of mean distance from sun for Pluto.

Planet	ECC	P YRS	P YRS	ERROR	a'
		Atlas	CALC	yr/kyrs	p^2/3
MOON	0.0500	0.0748	0.0747	1.73	0.0026
Hg	0.2056	0.2408	0.2313	39.51	0.3871
VENUS	0.0068	0.6152	0.6155	-0.51	0.7233
EARTH	0.0167	1.0000	0.9999	0.10	1.0000
MARS	0.0934	1.8810	1.8609	10.69	1.5238
JUPITER	0.0484	11.8620	11.8528	0.78	5.2012
SAT	0.0557	29.4580	29.3477	3.74	9.5383
URANUS	0.0472	84.0150	83.5974	4.97	19.1825
NEPTUNE	0.0086	164.7880	165.0150	-1.38	30.0573
PLUTO	0.2485	247.6970	235.3080	50.02	39.4409

4. LAGRANGE EQUATION FOR FLUID PASSING RADIALLY IN ANNULAR PASSAGE

Next, lets try the Lagrange equation for a fluid passing through an annular passage (such as a centrifugal pump impeller) in the radial direction between an inner radius of r_1 and an outer radius r_2. The kinetic energy of a fluid is $v^2/2g$, so fluid KE in a centrifugal field is

$\mathbf{T} = \{(dr/dt)^2 + (r\omega)^2\}/2g$ and define potential energy as $\mathbf{U} = (p/\rho)$, elevation head is neglected.

The Lagrange is then

$\mathbf{L} = \mathbf{T} - \mathbf{U} = \{(dr/dt)^2 + (r\omega)^2\}/2g - (p/\rho)$

$\partial \mathbf{L}/\partial r = \partial(½g\{(dr/dt)^2 + (r\omega)^2\})/\partial r_1 - \partial(p/\rho)/\partial r_1 =$

$\partial \mathbf{L}/\partial r = (1/g)\, r\omega\partial(r\omega)/\partial r - \partial(p/\rho)/\partial r_1 \Rightarrow (1/g)*r\omega^2 - \partial(p/\rho)/\partial r_1$ at constant ρ

$\partial \mathbf{L}/\partial V_r = ½g\, \partial\{(dr/dt)^2 + (r\omega)^2\}/\partial V_r = ½g\, \partial\{(V_r)^2 \}/\partial V_r \Rightarrow g\, V_r$

$d(\partial \mathbf{L}/\partial V_r)dt = d(V_r/(g))dt = (1/g)\, dV_r/dt$

The coordinate equations of motion based on $\{d(\partial \mathbf{L}/\partial Vr)/dt - \partial \mathbf{L}/\partial r \} = 0$ are:

$dV_r/dt - r\omega^2 + g\partial(p/\rho)/\partial r_1 \} = 0$ [the units =, ft/s^2, so, lets see}

The steady state equation reduces to $dp = \rho\omega^2(r_2^2-r_1^2)/2g$ which agrees with Eqn 2.9 M&S, via

$\partial(gp/\rho) =\omega^2 r\partial r \Rightarrow (g/\rho)dp = \omega^2 (\Delta r^2)/2 \Rightarrow dp = \rho\omega^2(r_2^2-r_1^2)/2g$

Lets try getting the total derivative in terms of partials

$dV_r = dr\,[\partial V_r/\partial r]+ dt\,[\partial V_r/\partial t] + d\theta\,[\partial V_r/\partial\theta]$ and the total derivative is

$dV_r/dt = [dr /dt]\, \partial V_r/\partial r + dt/dt\, \partial V_r/\partial t + d\theta/dt\,[\partial V_r/\partial\theta]$ For radial flow the last term is zero.

$dV_r/dt = V_r\partial V_r/\partial r + \partial V_r/\partial t \cong \Delta V_r^2/(2\Delta r) + \partial V_r/\partial t$ (which is velocity head)

The transform is made by $(dr/dt) = V_r$ & $VdV = \Delta V^2/2$ & finally

$\Delta V_r^2/(2\Delta r) + \partial V_r/\partial t - r\omega^2 + g\{\partial(p/\rho)/\partial r\} = 0$ and taking out ∂r with integration gives:

$\Delta V_r^2/2 + \Delta r(\partial V_r/\partial t) - \Delta(r\omega)^2/2 + g\{\partial(p/\rho)\} = 0$ **Lagrange Equation of r coordinate.**

Since $r\omega$ is tangential velocity, V_T, and $d(p/\rho)$ is just fluid head, the r equation is:

$\Delta(V_r)^2/2 + \Delta r(\partial V_r/\partial t) - \Delta(V_T)^2/2 + g\{head\} - \text{friction loss} = 0$ **Bernoulli Equation of Impeller**

The significance of the above Bernoulli impeller equation is in the dynamic term. Section 2.5 of the Pump handbook gives almost the same equation, but did not include the dynamic term. This equation states that dynamic head is equal to static head. If an imbalance happens, either by changing speed or by external changes to the head, then a dynamic effect happens, whereby flow changes. The purpose of this equation is to model the rate at which those flow changes happen. It is also just possible that this method will assist to predict instability points such as surge, minimum flow effects in pumps or provide a basis for making start-up calculations, when combined with the angular momentum equation. Typical to design of centrifugal impellers, is to lump the losses into an efficiency term, rather than calculate flow losses. Impeller designers also tend to calculate the net dynamic head by the Euler vector equations. The above equation quantifies the static terms with the dynamic terms.

5. THE LAGRANGE EQUATION FOR ANGULAR, θ, MOTION:

$\partial\mathbf{L}/\partial\theta = \partial(\tfrac{1}{2}g\{(dr/dt)^2 + (r\omega)^2\})/\partial\theta - \partial(p/\rho)/\partial\theta \equiv 0$ @ $p \neq f(\theta)$. The time derivative of θ is ω:

$\partial\mathbf{L}/\partial V_\theta = \partial\mathbf{L}/\partial\omega \ \tfrac{1}{2}g\ \partial\{(dr/dt)^2 + (r\omega)^2\}/\partial\omega = (1/g)r\omega\partial\{(r\omega)\}/\partial\omega = r\omega r = \omega r^2$

$d(\partial\mathbf{L}/\partial V_\theta)dt = d(\omega r^2/(g))dt = (1/g)\ [d(\omega r^2)/dt = (\omega dr^2/dt + r^2 d\omega/dt)] = [2r\omega(dr/dt) + r^2\alpha]/g$

$d(\partial\mathbf{L}/\partial V_\theta)dt = [r\omega V_r + (\tfrac{1}{2})r^2\alpha]/g$

$\{d(\partial\mathbf{L}/\partial\omega)/dt - \partial\mathbf{L}/\partial\theta\} = [r\omega V_r + (\tfrac{1}{2})r^2\alpha]/g = 0$ **Lagrange Equation of θ coordinate.**

The $r\omega V_r$, term is the torque per unit mass for the fluid at zero angular acceleration and the second term is just the torque/mass when subject to rotational acceleration of α. The term, $r\omega$, is the tangential velocity, V_t. The term $(\tfrac{1}{2})r^2$ is the mass moment of inertia, $(I = wk^2)$, per unit of mass for a thin disk. Since it is necessary to have a containment volume constructed of a material different than the fluid, the angular momentum equations for a solid disk will be developed later.

Adding these 2 Lagrange equations in units of fluid head: ft

$\{\Delta V_r^2/(2g\Delta r)\}dr - (r\omega^2/g)dr + (V_t V_r)/g + \{d(p/\rho)\} = \{(\partial V_r/\partial t)/g\}dr + [(\tfrac{1}{2})r^2\alpha]/g$

which when integrated at steady state for r gives $\{\Delta V_r^2/(2g) - \Delta V_t^2/(2g) + (V_t V_r)/g + \{\Delta(p/\rho)\} = 0$

This combined Lagrange equation for centrifugal head is basically just the classical head equation for a centrifugal impeller with incompressible fluid. The classical impeller is designed to have insignificant radial velocity at the inlet eye, so V_r at r_o = zero. When the tangential head is acting as a couple against the peripheral radial flow, the factor of ½ is dropped from the tangential head. The term $\Delta(p/\rho)$ is the expression for head developed, so the above steady state equation becomes:

$\Delta(p/\rho) = \Delta(\text{Head}) = V_{t2} [(\mathbf{V}_{t2}) - (\mathbf{V}_r)]/g = [(u)_2{}^2 - u_2Q/(A_p\tan\beta)]/g \equiv$ Eqn. 8-25 M&S

The term $1/\tan\beta$, arises from the vector addition.

The centrifugal head is related to the torque, T, by the power equation. Since power is

$T\omega = \rho Q(\text{Head})$, so $T = \rho Q(\text{Head})/\omega = \rho Q[V_{t2} [(\mathbf{V}_t)_2) - (\mathbf{V}_r)]/g\omega = \rho Q\, r_2(\mathbf{V}_{t2} - \mathbf{V}_{r2})/g$ by $(V_{t2}= \omega r_2)$

6. EULER METHOD

In order to better understand the second Lagrange equation, the Euler method will be applied. The moment of momentum equation is a fundamental equation for dynamic analysis of rotation. The principle states (Streeter 3.12): the resultant force acting on a control volume is sum of the net flux of momentum plus the rate of increase of momentum within the control volume.

$\Sigma\mathbf{F} = d(m\mathbf{v})/dt = \partial /\partial t\int\rho\mathbf{v}d\nu + \int\rho\mathbf{v}\bullet\mathbf{v}d\mathbf{A}$

Torque being the vector cross product of r & F, i.e. the moment of force about the center.

$r\bullet\mathbf{F} = T = d(m\mathbf{v}\bullet\mathbf{r})/dt = \partial\{\int\rho\mathbf{v}\bullet\mathbf{r}d\nu \}/\partial t + \int\rho r\bullet\mathbf{v}\bullet\mathbf{v}d\mathbf{A}$

The LHS is the net torque exerted by forces on the control volume. The RHS is the rate of change of moment of momentum within the control volume plus the net efflux from the moment of momentum from the control volume. This method is typically applied to turbo machinery where torque's are more important than forces.(Streeter p167) For the annular disk under consideration, the torque can be calculated from:

$T = \partial\{\int\rho\mathbf{v}\bullet\mathbf{r}d\nu \}/\partial t + \int\rho r\bullet\mathbf{v}\bullet\mathbf{v}d\mathbf{A} = \partial\{\int\rho v_t r d\nu \}/\partial t + \int\rho r V_t V_r d\mathbf{A}$

For an annular cylinder, the volume element is $d\nu = b\pi\, ((r+dr)^2-r^2) \cong b\pi(2rdr)$, and tangential velocity is $r\omega$, giving

$\int\rho v_t r d\nu = \int\rho\, r\omega r\, b\pi(2rdr) = (2\rho b\pi\omega)\int rr\,(rdr) = (2\rho b\pi\omega)\{(r_2)^4-(r_1)^4\}/4$

The annular volume = $(b\pi)\{(r_2)^2-(r_1)^2\}$ and density = mass/volume, which means the second term is just the basic acceleration torque term $I\alpha$, where I is the mass moment of inertia for the inner & outer radius, $I = m(½)(\{(r_2)^2+(r_1)^2\}$, based on $(a^2-b^2)(a^2+b^2) =\{(a)^4-(b)^4\}$

Streeter, Eqn 3.12.5, gives the last term to be $\rho Q[(rV_t)_2 - (rV_t)_1]$ but does not provide any derivation. In order to accept the results, it is necessary to check the validity. One way to make a check is as follows:

$Q= V_rA = V_r(b2\pi r)$ and ρ= mass/volume $= m/(b\pi)\{(r_2)^2-(r_1)^2\}$ so

$\rho Q = [V_r(b2\pi r)]m/[(b\pi)\{(r_2)^2-(r_1)^2\}] = [V_r(2r)]m/[\{(r_2)^2-(r_1)^2\}]$ and $V_t = \omega r$

$\rho Q[(rV_t)_2 - (rV_t)_1] = [V_r(2r)]m/[\{(r_2)^2-(r_1)^2\}][(r\omega r)_2 - (r\omega r)_1]$ & $\omega \neq$ function of r, so

$\rho Q[(rV_t)_2 - (rV_t)_1] = [V_r(2r)]m\omega$, combining this equation with the α equation gives

$T = [V_r(2r)]m\omega + m(½)(\{(r_2)^2+(r_1)^2\}\alpha$ so letting $(½)(\{(r_2)^2+(r_1)^2\} \cong (r)^2_{avg}$

$T/m = ftlb/lb = ft = \{[V_r(2r)]\omega + (r)^2\alpha\}/g$ which is identical to the Lagrange equation for θ. The term $½r^2$ is identified as I/m.

From which it is possible to see the Lagrange equation for θ coordinate is just the torque per mass of fluid, or angular acceleration.

7. TORQUE AND MASS MOMENT OF INERTIA

Let I be defined as the mass moment of inertia (#ft^2 also known as wk^2) & T be defined as the torque, ft#. Torque is the result of 1 pound force acting through a lever of 1 foot radius, i.e. one foot pound force. Typically in rotational problems, the wk^2 or I is considered as a mass, and torque as force, the basic relationships are given below. Torque's acting in the same plane, are added just as forces. In rotation, there will typically be a driving torque and a series of resisting torque's. For a power turbine, the driving torque is the motive power of the fluid, less the load and frictional torque's, plus, in case of speed changes, the inertial resistance factors. In the case of an electric motor, the driving torque is the result of electric power consumption. The net torque is the driving torque, less the resistant loads, such as friction, load requirements, and possibly inertia requirements when speed is changed.

The rotational Kinetic Energy = $\Delta KE = ½I(\omega^2_1 - \omega^2_0)$ 3

Torque, $T = I/g*d\omega/dt = I\alpha$ where g = 32.2 ft/sec/sec 4

In engineering terms, $308\Delta T/ wk^2 = \Delta RPM/\Delta t = dN/dt$ 4a

Power is the time rate of work $\equiv$ dU/dt . In the case of a body being acted upon by a force, F, and moving with a velocity, v, power is expressed as force times velocity. When a body is in rotation with an angular velocity, ω, and being acted upon by a torque, T, parallel to the axis of rotation, then power is defined as:

$dU/dt = T\omega$ in engineering units, HP = T*N/5280 & dU = dPOWER* dt = dU = $U_{1\rightarrow 2}$ 5

Using conservation of energy, $KE_1 + U_{1\rightarrow 2} = KE_2$ & common engineering units of horsepower, HP, gives

$\Delta KE = ½\ wk^2(N_1^2 - N_0^2)(2\pi)^2/32.2 = U_{1\rightarrow 2} = (\Delta t)550*60*60*HP_{avg}$ & collecting constants 6

$\frac{1}{2}wk^2(N_1^2 - N_0^2)(1274)^{-2} = (\Delta t) * HP_{avg}$ but $(N_1^2 - N_0^2)/2 = (N_1 - N_0)(N_1 + N_0)/2 \cong NdN$, then 7

$NdN/dt = HP_{avg} / wk^2 * (1274)^2$ 8

Equation 8 can also be validated by algebra combination of equations 4 & 5. One consequence of Eqn.8 is that zero speed has a singular point of zero flow at zero net torque. This point has been validated for rotating hydraulic equipment by R. T. Knapp, ASME Trans., Nov. 1937. Equation 8 is generally used in studies of operating pumps and turbines under non-steady flow conditions. Equation 4 is generally used in studies of starting pumps and turbines under non-steady flow conditions.

Typically the value of I will be given by the equipment manufacturer. The value is determined by observing the time for the rotor to come to rest from various speeds. The slope of $\Delta RPM/\Delta t$ is the value of wk^2 and the intercept is the friction resistance torque, Eqn 4a. In absence of data, the NEMA normal wk^2 for a motor can be calculated from $BHP^{1.15}/(RPM/612)^2$, Ludwig. Coupled pump motor systems are typically 300+% of the NEMA normal wk^2 . For simple shapes, the value of I is given in the table below:

SHAPE	$I = W k^2$
thin solid disk or cylinder about radius	$\frac{1}{2} mr^2$
circular wire or hoop about radius	mr^2
sphere	$^2/_5\ mr^2$
cylinder about ends	$^1/_{12}\ m(3r^2 + L^2)$

The friction of rotational equipment consist of disk friction in the case of centrifugal impellers, bearing friction, seal friction, static friction, and sliding friction. The coefficient for sliding friction is typically less than the coefficient of static friction. For static friction, in many cases is taken as a fixed percent of the operating torque requirement, between 10 and 15%, and decreases to zero at 15 to 20% of normal speed (p2-164 pumps). One formula given for sliding friction is fluid horsepower to 0.4 power, this works to be about 6% of normal power requirements. The friction working in opposition to rotation should not be confused with friction that opposes fluid flow. Friction acting against fluid flow, acts to reduce available fluid energy, and is entered as an efficiency factor to the fluid power. Fluid friction enters into the torque equation based on either, available energy for work for turbines, or as input energy required to reach a specified power level for a pump.

A pump or compressor torque speed curve is then built up based on the pump affinity laws, see pumps p.2-165 for more details. In general there are 2 cases, operation against a check valve and operation against friction and momentum resistance. The more common case is operation against a check valve. The torque up to the check valve opening is based on fluid torque being proportional to speed squared. Once the check valve opens, the system resistance vs flow curve needs to be calculated based on considerations of non-steady state flow. The easiest way is to add an acceleration head to the conventional Bernoulli equation.

The single tube, center pivot, Garden Sprinkler was considered by Streeter as example 3.26. The solution by Streeter was based on the Euler control volume integration. It is intended to present Streeter's solution and the Lagrange method as a comparison. The problem considers the sprinkler discharging water upward and outward from the horizontal plane so that the discharge is at an angle of θ^o to the horizontal datum. It has a constant cross sectional area A_o and discharges q cfs starting with w = 0 at t_o. The resisting torque from bearings and seals is considered constant as T_o and the moment of inertia of the rotating empty sprinkler head is I_s, from which Streeter determines the net torque to be equal to the frictional torque:

$T_o = \partial\{\int\rho \mathbf{v \bullet r} d\nu\}/\partial t + \int \rho r \bullet \mathbf{v \bullet v} d\mathbf{A} = \partial\{\int \rho v_t r d\nu\}/\partial t + \int \rho r V_t V_r d\mathbf{A}$

$\int \rho r V_t V_r d\mathbf{A}$ **=** Was not integrated in the Streeter example, however a rational approach is to consider just the torque developed from the flow couple and net tangential velocity. Furthermore in single plane torque's, the general equation of motion is $T = I/g{*}d\omega/dt = I\alpha$. The net torque is accelerating torque less the friction torque. The total I, is the fluid mass moment plus the empty sprinkler head I_s.

$T_{fluid} = 2\rho Q r(\mathbf{V}_t - \mathbf{V}_r) = 2\rho(q/2)r_o(V_r Cos\theta - \omega r_o)/g$, $\{V_r Cos\theta$ is the tangential component of $V_r\}$

$T_{fluid} = \rho(q)r_o(V_r Cos\theta - \omega r_o) = \rho(V_r(A_o))r_o(V_r Cos\theta - \omega r_o)$ since $V_r(A_o) = q$

$T_{fluid} = \rho(A_o)r_o(V_r V_r Cos\theta - V_r \omega r_o)$

$-T_o = 2\partial\{\int_0^{ro}(\rho v_t r A_o dr\}/\partial t + I_s\alpha - \rho q r_o(V_r Cos\theta - \omega r_o)$

$-T_o = 2\partial\{\int_0^{ro}(A_o\rho\omega r^2 dr\}/\partial t + I_s\alpha - 2\rho q r_o({}^1/_2 V_r Cos\theta - {}^1/_2\omega r_o)$

$-T_o = 2A_o\rho\alpha r^3/3 + I_s\alpha - 2\rho q r_o({}^1/_2 V_r Cos\theta - {}^1/_2\omega r_o) = \alpha({}^2/_3 A_o\rho\ r_o^{\ 3} + I_s) - \rho q r_o(V_r Cos\theta - \omega r_o)$ &

$d\omega/dt\ ({}^2/_3 A_o\rho\ r_o^{\ 3} + I_s) = \rho q r_o(V_r Cos\theta - \omega r_o) - T_o$ (Streeter's Equation)

However an improvement is to note that ${}^2/_3 A_o\rho\ r_o^{\ 3}$ is I_w for the water inside the rotating tube, since the water mass is

$\rho{*}2r_oA_o$. I_w/m for a thin rod is $L^2/12 = (2\ r_o)^2/12 = 4\ r_o^{\ 2}/12 = {}^1/_3 r_o^{\ 2} \Rightarrow I = {}^1/_3 M\ r_o^{\ 2}$

Letting M be the combined mass of the sprinkler rod plus water inside the rod gives

$d\omega/dt\ (I_w + I_s) = d\omega/dt\ ({}^1/_3 M\ r_o^{\ 2}) = \rho q r_o(V_r Cos\theta - \omega r_o) - T_o = r_o\rho(2Ao)V_r(V_r Cos\theta - V_t)/g - T_o$

Verify results using the Lagrange method. The last term is the fluid energy term from the steady state Lagrange Equation of head.

$\{\Delta(\ p/\rho\ )\} = \Delta V_r^{\ 2}/(2g) - \Delta V_t^2\ /(2g) + (V_t V_r\ )/g$

In this case, it is assumed that all potential or head energy is translated to kinetic energy. It is also noted that Streeter simplified the equation by not including the acceleration head. This neglect is likely ok, however, energy expended on acceleration would reduce energy available to rotate the mass. Having stated that radial velocity is constant over the tube length reduces ΔV_r to zero, and the inlet V_t is zero, for a point source entering at the center, leaving:

Input Head = : $V_t^2\ /(2g) - (\mathbf{V}_t \bullet \mathbf{V}_r\ )/g = V_{t2}({}^1/_2 V_r Cos\theta\ - {}^1/_2 V_{t2})/g$ (${}^1/_2 Cos\theta$ comes from vector dot product)

Since power is $T\omega = \rho Q(\text{Head}) = \rho 2AV_r(\text{Head})$ due to the 2 nozzles involved in flow, fluid torque is

$T = \rho Q(\text{Head}) = \rho 2AV_r(\text{Head})/\omega = \rho 2AV_r(V_{t2}(^1/_2V_r\cos\theta - {}^1/_2V_{t2})/g)/\omega = \rho AV_r(r_2(V_r\cos\theta - V_{t2})/g)$

The net torque for acceleration is fluid torque less friction torque and so

$(I/g)d\omega/dt = \rho AV_r(r_2(V_r\cos\theta - V_{t2})/g) - T_f$ [(I/g) is used because torque is ft-lb.]

8. CALCULATION OF PRESSURE TRANSIENTS IN GAS LINES

The boundary conditions were taken as follows: inlet mass flow was dependent on the speed of recycle valve closure with an initial spike of 25 mmscfd. The exit pressure was to the inlet of the Abqaiq-Berri gas line at GOSP 6. This boundary condition was considered constant at 450 psig, irrespective of gas flow rate, due to PCV-103, at GOSP-6 bleeding off gas to Shedgum.

The partial differentials were taken as whole differentials because corrections for pipe volume were dropped due to anticipated low pressure changes. The minor pressure changes also provides for the assumption that gas density depends only on pressure, as explained for equation 4, below.

$$\partial G/\partial t = -144g/L\{\Delta(\text{psi}) + \Delta G^2/(2\rho g*144) - \Delta p_f\} \qquad 1$$

$$\partial\rho/\partial t = \{G_2(t) - G_1\}/L \qquad 2$$

Equation 1 calculates the outlet mass velocity out at point 2 given the values of the terms. Equation 2 calculates the upstream pressure transient, given the initial condition of P at the downstream boundary and G at the upstream point. At any given time step the equations are solved for the two unknown values of mass velocity out and the upstream density. The pressure is then calculated from the density by equation 3. The pipe volume is taken as constant, i.e. no elasticity effects on the pipe wall.

$\rho = PM/RTZ$ which can also be arranged to calculate pressure as $P = \rho\, RTZ/M$ 3a/b

The friction pressure drop is calculated as: $\Delta p_f = 43.48\{w|w|\}(f*L_e)/\{\rho * d^5\}$ psi 4

Equation 4 uses the time dependent density at the inlet point and the boundary condition flow rate. The frictional pressure loss typically changed from zero to 11 psi as the line was flow packed. The initial pressure was taken at 465 psia and since the pressure loss was less than 10% of the initial pressure, the use of equation 4 is a valid means of determining the pressure drop. The flow was considered isothermal and adiabatic due to the low pressure drop plus the gas temperature was also approximately ambient temperature.

The friction factor was considered constant and was based on 110% of the fully turbulent value. The approximation was made by equation 5 as given below:

$f = 1.1*\{1.14 - .86*\ln(\varepsilon/D)\}^{-2}$ with $\varepsilon = .000165$ and $D = d/12$ 5

The solution method for the water hammer differential equations was tested using example problems from V.L. Streeter for both liquid water hammer and for damped Utube oscillations. This test was made to verify accuracy the ODE solver. Those results were promising and so the ODE solver was then applied to the above equations.

The simulation results are provided in Graph XX below. The results indicate that an initial pressure transient of 2 psi would be generated from the given boundary conditions. However, to improve modeling results, it would be necessary to consider the line pack of the entire AB-BGP line and whether the line will be operated on flowing or back pressure control at the Berri end. The PFD's for BI-3130 indicate that backpressure at Abqaiq will be maintained via PCV-103, at GOSP 6 slug catcher. If ABGP-1 were not packed for flowing pressure, then the constant pressure boundary condition would need changing and a more detailed simulation produced to account for the line pack from Abqaiq to Berri.

The concern raised by this simulation was that the pressure transient of 2 psi exceeded the normal pressure drop for the check valves. Should such a transient happen in practice depends on the closing characteristics of the selected check valve. However should a rapid open and close action happen with the check valve, then the possibility of severe and destructive harmonic vibrations may be a concern.

These preliminary results along with the Mokveld flow/pressure drop curves for the non slam check valves will require additional analysis, pending detailed information on the dynamic factors for the non slam check valves. However, the vendor has been pressed to provide such information. One of the points why dynamic effects require additional attention from vendors. Attached are simulation results for various closure times of the recycle valve. The units of the above equation are:

- $G \equiv$ lb/sec/sf = lb/sec/(inside pipe flow area)
- $g \equiv$ 32.2 ft/sec/sec
- d = inside pipe diameter, inches 23"
- ρ = density lb/cf Calculated value
- P = pressure in psia (boundary at 465psia, other is dynamic)
- M = molecular weight 32.7
- R = gas constant 10.73
- T = temperature in Rankine, 460 + F 150F used, as a constant value
  During startup the temperature actually changes with time
- Z = gas compressibility factor 0.9 (an additional run should be made w/ 0.8)
- L = Length of pipe for volume calculations 12,000'
- Le = Equivalent Length of pipe to account for friction effects. 15143'
- w = mass rate lb/sec 150 maximum, 25 initial spike
- Δpsi = pressure head change upstream pressure(t) - pressure downstream
- $\Delta G^2/(2\rho g*144)$ = Velocity head, psi (static head taken as zero)

9. DYNAMIC EQUATIONS FOR A SIMPLIFIED CENTRIFUGAL PUMP WHEEL

Per section 8, the dynamic equations for a simplified centrifugal pump wheel are developed. It is the intent of this article to present the dynamic equation of a centrifugal gas compressor wheel. The basic equations are presented again:

$[r\omega V_r + (½)r^2\,\alpha]/g = 0$ **Lagrange Equation of θ coordinate.**

$\Delta V_r^2/(2\Delta r) + \partial V_r/\partial t - r\omega^2 + g\{\partial(p/\rho)/\partial r \} = 0$ **Lagrange Equation of r coordinate.**

For constant speed operation, the Lagrange Equation of θ coordinate may be dropped. The head equation for radial flow is then:

$\Delta V_r^2/2 + \Delta r\,(\partial V_r/\partial t) - \Delta(r\omega)^2/2 + g\{\Delta(p/\rho) \} = 0$

The last term is just the head, which is integrated for the adiabatic conditions of p/ρ^k = constant and density, ρ = PM/RTZ, to be

$\Delta V_r^2/2 + \Delta r(\partial V_r/\partial t) - \Delta(r\omega)^2/2 + g\{1544Tz/Mwn[(p_2/p_1)^n - 1] \} = 0$

The other terms have been previously defined, n is just the polytropic exponent, $(k\text{-}1)/k\varepsilon_p$ The Radial continuity equation is arrived at from conservation of mass: rate of mass in less rate of mass out equals accumulation or

$\{(dm/dt)_{|ro+dr} - (dm/dt)_{|ro} \}(dt) = (\rho_{|to+dt} - \rho_{|to})\upsilon$ & $\{(G^*A)_{|ro+dr} - (G^*A)_{|ro} \}(dt)= (\rho_{|to+dt} - \rho_{|to})\upsilon$

For the annular area and volume,

$A_{|ro+dr} = b(2\pi)(r+dr)$ & $A_{|ro}= b(2\pi)r$ & $\upsilon = b(\pi)\{(r+dr)^2-r^2\} \cong b(\pi)\{(2rdr) \}$, so

$\{ b(2\pi)(r+dr)(G^*)_{|ro+dr} - b(2\pi)r(G)_{|ro} \}(dt)= (\rho_{|to+dt} - \rho_{|to})\, b(\pi)\{(2rdr) \}$ & division by $b(\pi)(2rdr)(dt)$ gives

$\{ (r+dr)/rdr(G^*)_{|ro+dr} - \{(G)_{|ro} \}/dr = (\rho_{|to+dt} - \rho_{|to})/(dt)$

$\{ (1/dr+1/r)(G^*)_{|ro+dr} - \{(G)_{|ro} \}/dr= d\rho/dt$

$\{dG/dr + (1/r)\,(G)_{|ro} \} = d\rho/dt$

$\partial\rho/\partial t =\{G_2(t) - G_1\}/\Delta r + (G/r)_1$

At steady state, this reduces to dG/G = -dr/r or G2/G1 = r1/r2, which is correct, since G(2πrb) = dm/dt, which is constant at steady state. Where G is the mass velocity and Δr is the distance from the compressor inlet eye to the outlet of the wheel. These 2 equations cover the surge conditions for the compressor wheel. However owing to the very short distances of a compressor wheel, the simulation step time will be on the order of 1 millisecond to accurately calculate the surge wave.

The attached graph is the dynamic version of Streeter Ex.9.9, a constant speed compressor wheel.

10. CONTROL VALVE MODELS:
The Control valve model used for gas is the Fisher valve model. The reason for using this equation is that it is a simple way to ensure flow does not exceed critical flow values. Another factor is that, when mass rates are expressed in pounds per second (pps), the numerical values of pps are identical to MMSCFD for a gas of mol weight of 32. Most of the off gas from Abqaiq is about 32 mol weight, which makes simulation units easy to relate with operating data.

The Fisher Equation is:
$pps = 1.1(60)^{-2}C_g\sqrt{(\rho P_1)}\ \sin\{\sqrt{(\Delta p/P_1)}\ 59.64/C_1\}$

Where P_1 is inlet psia, ρ is inlet density in pcf, Δp is pressure difference across the valve in psi, The values of C_1 & C_g can be obtained from valve sizing calculations, vendor datasheets, or alternatively from Fisher Valve book, or Figure 4.32 of the GPSA. The bracket quantity is a radian angle, in case this must be converted to degrees, divide by $2\pi/360$. The bracket quantity should not exceed 90^o,or $\pi/2$ radians, otherwise the valve reaches critical flow. Critical flow causes excessive pipe vibrations and noise . Under critical flow conditions, flow is varied only by changing the upstream pressure, not by the total dp. Critical flow should be avoided in a control valve. An important aspect of this equation is that $C_g = C_1 * C_v$. Many valve vendors only quote the valve C_v values. Tables 1.7 and 1.6 of Process Engineers' Hdbook provide a list of typical C_1 values. A summary is given below:

TYPICAL VALUES OF C1

Type	Cage Globe	Ball, V, modify, etc	Ball Full area	Butterfly	Butterfly V type
Typical C_1	33 to 29	23	20	24.7	16

For liquid control valves, the valve flow performance is determined from gpm = $Cv/\sqrt{sg}$. (Control Valve Handbook, Fisher Controls p62, 2nd Ed 1977)

In either case, the flow capacity is varied by ranging the PID controller output between zero and 1. This output is then run into the valve characterizer which can be either, linear, equal percent, butterfly type, or quick opening. The output from the valve characterizer is then multiplication time the maximum Cv to determine valve flow rate. The Perry equation to characterize the valve is given below. With a $\cong$ 0.1, for equal percent, others, as suitable for the trim, CO is controller output, between zero and one. The attached graph shows the characteristics of various trims.

$Cv = Cv\ max\ times\ [(CO)^2/(a-(1-a)(CO)^4)^{0.5}$

Every control loop will have a number of dead times. In the case of sensing instruments, pressure and differential pressure loops are relatively fast one to two seconds, while temperature elements have response times on the order of 10 to 15 seconds. Dead times a modeled by means of a first order transfer function to determine response as a function of time. For valve response times, antisurge valves are typically sized to have between 2 and 4 seconds open/close time cycle times. Other control valves typically have stroke times between 10 and 20 seconds, depending on valve size and actuator size. In addition to stroke response times, a valve can have slip or deadband characteristics. The factors can be modeled by use of time delay and first order filters to the input. (Improve Control Valve Response, CEP pp76-84, June 95, G.K. McMillan)

11. OTHER VALVE ELEMENTS

Valves serve as a major boundary point in dynamic simulation and plant operation. Therefore it is desired to define the dynamic operation of valve elements with sufficient accuracy to properly control the actual process, this also assist in development of a sufficiently detailed model.

A major consideration for an isolation valve is rate of opening and rate of closing. Additionally, the subsequent flow response under these conditions is an important consideration. For MOV and AOV valves engineering standards set the operational rate at 12" per minute for gate valves and ball valves, ie a 6" valve closes in 30 seconds. During the operation of an isolation valve, the flow through the valve can be predicted, either based on a control valve model, or based on an empirical model. The two empirical models presented by Streeter are:

$V(t) = V_o \tau \sqrt{[Ht/Ho]}$ & $\tau = (1-t/t_c)^3$ or $\tau = 1-(t/t_c)^{1/3}$

For check valves M&E p222, recommend a minimum closure time of 10 seconds, being about ½ closed when velocity reaches zero or an adjustable closure time between T & 4T. Where T= LV/gH, H being the average de-acceleration head, including friction.

For check valves and relief valves, other authors have presented models to define the dynamic behavior of these valves. For a spring loaded valve the Lagrange equation can be defined as:

$d(\partial \mathbf{L}/\partial V)/dt - \partial \mathbf{L}/\partial X = 0$ & $\mathbf{L} = \mathbf{T} - \mathbf{U}$

V is coordinate velocity, dX/dt best considered as a variable
X is coordinate
T is kinetic energy
U is potential energy $\partial \mathbf{U}/\partial X = \mathbf{F}_x$

$Us = \int F dx = \int kX dX = ½KX^2$,

$\mathbf{U} = ½(k_a[X]^2 + mgX$ (X being the vertical displacement) & $\mathbf{T} = ½(m[V]^2)$

$\mathbf{L} = \mathbf{T} - \mathbf{U} = ½(m[V]^2) - [½(k_a[X]^2 + mgX + \int \mathbf{F} \bullet \mathbf{d}x]$

$\partial \mathbf{L}/\partial X = -k_a[X] - mg - \Sigma \mathbf{F}_x$ and $\partial \mathbf{L}/\partial V_x = m[V]$

$d(\partial \mathbf{L}/\partial V)dt = m[dV]/dt = m[d^2X]/dt^2$ Finally the equation of motion is: $d(\partial \mathbf{L}/\partial V)/dt - \partial \mathbf{L}/\partial X = 0$

$m[d^2X]/dt^2 + k[X] + mg + \Sigma \mathbf{F}_x = 0$

The additional key forces identified by Kruisbrink, (Kruisbrink, A. C. H. Chapter 11 Proceedings, 3rd International Conf. On Valves & Actuators, STI, Oxford UK), are: spring set; damping; friction; backpressure; front; his final equation is:

$m[d^2X]/dt^2 + k[X] + mg + S_o + C(dX/dt) + C_f A_d(\text{sign } dx/dt) + P_2 A_d - \Sigma \mathbf{F}_f = 0$

$\Sigma \mathbf{F}_f = P_1 A_n + [P_1 + ½\rho(V_1^2 - V_j^2)](A_d - A_n) + (P^*A)_{seat} + \rho[(V_1 - dx/dt)A_n](V_1 - V_j)\cos\gamma$

Auxiliary equations are: $[dV_n]/dt = d^2X/dt^2$. Minimum dp = set pressure = $(S_o + mg)/A_n$

$Q_{cv} = (V_1-(dX/dt))A_n = (VA)_j$ continuity

$V_j = [Q -(dX/dt))A_n]/[\mu\pi D_n X\sin\gamma$ where μ flow contraction coefficient.

$V_j = \sqrt{[2(P1-Pv)/ \rho + V_1^2]}$ if $P_j < P_v$ & $X_1>0$, @ X=0, $P_j=P_2$ cavitation parameter

The paper gives details of the constants for a 3K4 liquid relief valve with spring for a set pressure of 47 M of water, 6 bar. The above equation is valid for vertical valves, when valve is horizontal mounted, the gravity term, mg, should be dropped. A variety of valves function on this basis; automatic recirculation valves, non-slam check valves, and conventional relief valves. Pilot operated valves function based on pilot areas, and so the spring forces must be replaced by pressure times area. Pilot operated valves are also susceptible to harmonics, only to a lesser degree than spring operated valves. For gas phase flow, the equations of flow need to consider critical or non critical flow with variable density.

12. LAGRANGE METHOD TO PARTICLE FALLING IN FLUID:

For a particle of density, ρ_s, in free fall in a media of density, ρ_f, the KE & PE terms are:

$\mathbf{T}$ =KE = $½mV^2 = ½\rho_s\upsilon V^2$ & $\mathbf{U}$ =PE = $(\rho_s - \rho_f)\upsilon gX$, where υ = volume = $^4/_3\pi r^3 = {}^1/_6\pi d^3$

$\mathbf{L} = \mathbf{T} - \mathbf{U} = ½\rho_s\upsilon V^2 - [(\rho_s - \rho_f)\upsilon gX + \int \mathbf{F}\bullet d\mathbf{x}]$

$\partial\mathbf{L}/\partial X = -(\rho_s - \rho_f)\upsilon g - \Sigma\mathbf{F}_x$ & $\partial\mathbf{L}/\partial V_x = \rho_s\upsilon[V]$

$d(\partial\mathbf{L}/\partial V)dt = \rho_s\upsilon[dV]/dt = \rho_s\upsilon[d^2X]/dt^2$

Finally the equation of motion is: $d(\partial\mathbf{L}/\partial V)/dt - \partial\mathbf{L}/\partial X = 0$ is $\rho_s\upsilon[d^2X]/dt^2 + (\rho_s - \rho_f)\upsilon g + \Sigma\mathbf{F}_x = 0$

Here, the force, $\mathbf{F}_x$, acts in opposition to the acceleration of gravity. This force is the drag force;

$\Sigma\mathbf{F}_x = C_dA(½\rho_sV^2)$, A, is area normal to flow, substitution gives the equation of motion as:

$\rho_s\upsilon[d^2X]/dt^2 + (\rho_s - \rho_f)\upsilon g - C_dA(½\rho_sV^2) = 0$ with A of sphere = $½ (4)\pi r^2 = {}^1/_4\pi d^2$

At steady state, this reduces to Newton's Law of settling for spherical particles:

$(\rho_s - \rho_f)\upsilon g = C_d\bullet A(½\rho_fV^2) = C_d{}^1/_4\pi d^2(½\rho_fV^2) = (\rho_s - \rho_f){}^1/_6\pi d^3g \Rightarrow V_t= [4gd(\rho_s-\rho_f)/3C_d\rho_f]^{0.5}$

$C_d = 24/N_R + 3/\sqrt{N_R} + 0.34$; for sphere only & at the upper limit $C_d = 24/N_R$ gives stokes law. The correct value of velocity is the relative velocity, particle velocity, less fluid velocity.

The above equation for C_d based on Reynolds number, N_R the upper limit of validity is about N_R =100,000. For disks in free fall, at $N_R > 1000$, the C_d =1. In analyzing this type of motion, it is necessary to discuss drag and lifting coefficients. The best examples are airfoils or a car body. The drag is a resisting force acting against the direction of motion. The lift acts perpendicular to motion. Both lift and drag are calculated based on the forward velocity, but different area. In the case of drag, the correct area is the projected area, perpendicular to the flow. For lift, the correct area is area parallel to flow. In the case of an wing section, the lift area is length by width, and the drag area is length by height. Newton's method for calculation of C_d drag is to equate C_d to $2\sin^2\theta$. So for a plate, the max value of Cd is θ equal to 2 at 90 degree, and as the plate is rotated parallel to the flow, the drag approaches zero for a very thin plate. The lift force also decreases to a minimum along with the drag force and also lift increases with the pitch angle. When the lift and drag forces become equal, this condition, for an airfoil, is stall; typically between 30 and 45 degree.

13. CHECK VALVE MOTION

Considering a ball type check valve, seated in a holder of diameter only slightly less than the ball. The ball is caged inside a vertical pipe, the pipe diameter being greater than the ball diameter. With both areas equal, flow will not be established, until the pressure upstream exceeds the downstream pressure, $\Sigma \mathbf{F}_x = (PA)_u - (PA)_d$. If the upstream pressure increases just above the downstream static pressure, then flow will be established, causing an additional decrease in d/s pressure due to the friction losses across the ball. Next, there are 2 positions the ball achieve. If the fluid velocity between the pipe is ball and the wall exceeds the terminal velocity of the ball, then the ball will be accelerated to the top of the holder cage. If the average fluid velocity is less than the terminal velocity, then the ball will fall to the height at which the vertical fluid velocity equals the terminal velocity, owing to decreased annular area. Taking an arrangement to produce a 45^o flow angle, the height would be based on $V_t = Q/(0.707\pi Dh)$. Should flow stop, the time for a seal to be established would be nearly h/V_t. In the case of caged vertical valves, drag and lift are acting parallel.

14. FLAPPER TYPE CHECK VALVE

Here the calculation become more involved, due to the changing projected area and the fact that gravity forces cannot act parallel to the flapper. A vertical flapper would stick upright once flow stopped, due to lack of gravity action. Hence, most flapper type valves are hinged at between 45^o and 90^o inside a horizontal pipe. For a check valve, the drag force is the force acting to pull the disk away from the hinge, and lift is the force acting to raise the flow stop device. Considering the 90^o flapper type, with θ, being the angle from the vertical, then

$$\tfrac{1}{2}m\,dv/dt - (\rho_s - \rho_f)\upsilon \bullet g \bullet h - F_{back} - F_{forward} - F_{lift} - F_{drag} - F_{friction} = 0,$$

For full open position at steady state the fluid lift force = Buoyant Force

$$C_L A(\tfrac{1}{2}\rho_f V^2)/g = ((\rho_s - \rho_f)Az \Rightarrow (2gz(\rho_s - \rho_f)/(C_L) = \rho_f V^2 = C_c^2,\ \text{lb/ft/s}^2$$

The C_c stands for the Crane constant for full lift velocity, Crane Tech Paper 410, pA27. The constant is defined in terms of $\beta^2 V\sqrt{\rho} = C_c$, so squaring both sides gives $\beta^4 V^2 \rho_f = C_c^2$. The term β, is the ratio of pipe diameter. One worked example is given below and others results are in Table format. In the case of flow acting perpendicular to the surface, then the full lift C_c is for the drag coefficient, not the lift coefficient. Since for most fluids and steels, $(\rho_s - \rho_f)$ is about 9lb/sf, with z= ¼ inch plate, gives: For a swing type check valve, C_c=35, (other given below), so: $9(64.4)/C_L = 35^2 \Rightarrow C_L = 0.473$

TYPE	C_c/β^2	C_L
Swing Check Straight	35	0.473
Swing Check Globe Bowl	60	0.161
Swing Check Globe Bowl range (48-100)	48	0.252
Tilt Disk 5 degree stop (zero = Horizontal	80	0.090
Tilt Disk 15 degree stop	30	0.643
Lift Check 45 degree slope	140	0.030
Lift Check Vertical	40	0.362
Footer poppet type	15	2.573
Footer Hinged Lift Disk type	35	0.473

15. FLAPPER TYPE CHECK VALVE EQUATION OF MOTION:

Since there will be no movement in the radial direction, it is best to consider kinetic energy based on rotational velocity, ω.

T =KE = $\frac{1}{2}I\omega^2$ & **U** =PE = $(\rho_s - \rho_f)\upsilon gh$, where υ = volume = At

$$\mathbf{L} = \mathbf{T} - \mathbf{U} = \tfrac{1}{2}I\omega^2 - [(\rho_s - \rho_f)\upsilon\, g\, r\, (\sin\theta) + \int \mathbf{F}\bullet\mathbf{r}\, d\theta]$$

$$\partial\mathbf{L}/\partial\theta = (\rho_s - \rho_f)\upsilon g r(\cos\theta) - \Sigma\mathbf{F}_\theta\bullet\mathbf{r}$$

$$\partial\mathbf{L}/\partial\omega = I\omega$$

$$d(\partial\mathbf{L}/\partial\omega)dt = d[I\omega]/dt = \rho_s\upsilon\, r^2[d^2\theta]/dt^2$$

Finally the equation of motion is: $d(\partial\mathbf{L}/\partial V_\theta)/dt - \partial\mathbf{L}/\partial\theta = 0$

$$I[d^2\theta]/dt^2 + (\rho_s - \rho_f)\upsilon g r(\cos\theta) - \Sigma\mathbf{F}_\theta\bullet\mathbf{r} = 0$$

Here, the forces, $\Sigma\mathbf{F}_\theta$, can be considered as the following, lifting force (acting in opposition to gravity), the damping force (acting in opposition to the movement), and the static pressure force (only a factor when the disk is closed, $\{(A\,P)_u - (AP)_d)\}$). This force is the fluid acting on the disk.

$$\Sigma\mathbf{F}_\theta\bullet\mathbf{r} = [C_L A(\tfrac{1}{2}\rho_f V^2) + C_d A(\tfrac{1}{2}\rho_f V^2) +]*r*\cos\theta$$

A, is area of disk, possibly, $\pi r^2 = \frac{1}{2}\pi d^2$, substitution gives the equation of motion as:

$$I[d^2\theta]/dt^2 + (\rho_s - \rho_f)\upsilon g r(\cos\theta) - [C_L A(\tfrac{1}{2}\rho_f V^2) + C_d A(\tfrac{1}{2}\rho_f V^2)]*r*\cos\theta = 0$$

$$I[d^2\theta]/dt^2 + \{(\rho_s - \rho_f)\upsilon g - C_L A(\tfrac{1}{2}\rho_f V^2) - C_d A(\tfrac{1}{2}\rho_f V^2)\}\, r\cos\theta = 0$$

Which agrees with the above for full lift position. The velocity for the lifting force, is the velocity of fluid inside the pipe and the damping velocity is taken as $\omega = d\theta/dt$ to give:

$$I[d^2\theta]/dt^2 + \{(\rho_s - \rho_f)\upsilon g - C_L A(\tfrac{1}{2}\rho_f V^2) - C_d A(\tfrac{1}{2}\rho_f\omega|\omega|)\}\, r\cos\theta \; -(\mathrm{sign}\,\omega)\, T_{friction} = 0$$

Additional auxiliary equations for solving this position equation, are x =r cosθ, y = r sinθ, and the range allowed for rotation in terms of θ, ω = dθ/dt, plus initial values and the value of, C_L, lift factor as determined above and the drag damping coefficient, plus the friction torque. If the check valve is either a horizontal spring type or a ball type, the analysis is better in linear coordinates. A plot for a swing check is given for an assumed weight and Cd.

16. PID CONTROLLER

The PID controller is another major component of the process model. PID controllers can be set up with feed forward, set-point reset, or split range. The basic equation used for the Vissim PID is

Controller Output = $\Sigma\{K_p(PV-SP) + K_i\int(PV-SP)dt + K_d d(PV-SP)/dt)\}$

The constant, K_p, of the right-hand side is the proportional band, the constant, K_i , is the integral constant, and the third constant, K_d , is the derivative constant. When the difference between the process variable, PV, and the setpoint, SP, there is no change to the summation and the controller output remains constant. Additional discussion on the use of PID controllers and determination of the gain constants can be found in section 4 of the GPSA.

In dynamic simulation, it is the responsibility of both process model and the equipment model to determine how the process variable responds to changes in the operational parameters.

17. DYNAMIC SIMULATION OF FLUID FLOW WITH HEAT EXCHANGE

The following is a simplified routine for determining the temperature a pipe exchanging heat to a constant temperature atmosphere. It assumes the pressure head loss is low enough to neglect temperature changes from gas expansion. It also assumes the pipe wall has infinite thermal conductivity, thereby eliminating any radial temperature variations.

Heat in = $[Wc_p]_f(T_i-T_r)d\theta$ & Heat out = $\{[Wc_p]_f(T_o-T_r) + UA(1/2(T_i+T_o)-T_E)\}d\theta$

Accumulation = in - out = $\{[Wc_p]_f(T_i-T_o) + UA(T_E - {}^1/_2(T_i+T_o))\}\ d\theta$

Accumulation = $dT_o\{(\rho\pi r^2\Delta Xcp)_f + (\rho 2\pi rt\Delta Xcp)_m\}$ solving the equality for accumulations:

$$dT_o/d\theta = \{[Wc_p]_f(T_i-T_o) + UA(T_E - {}^1/_2(T_i+T_o))\}/\ \{(\rho\pi r^2\Delta Xcp)_m + (\rho 2\pi rt\Delta Xcp)_p\}$$

$$dT_o/d\theta = \{[Wc_p]_f(T_i-T_o)+U(2\pi r\Delta X)(T_E - {}^1/_2(T_i+T_o))\}/\ \{(\rho\pi r^2\Delta Xcp)_f+(\rho 2\pi rt\Delta Xcp)_m\}$$ & division by $2\pi r\Delta X$

$$dT_o/d\theta = \{[Wc_p]_f(T_i-T_o)/(2\pi r\Delta X) + U(T_E - {}^1/_2(T_i+T_o))\}/\ \{({}^1/_2\rho rc_p)_f+(\rho tc_p)_m\}$$

$$dT_o/d\theta = \{[W(c_p\ (T_i-T_o)-\Delta V_{head}]_f\ /(2\pi r\Delta X) + U(T_E - {}^1/_2(T_i+T_o))\}/\ \{({}^1/_2\rho rc_p)_f+(\rho tc_p)_m\}$$

Where the subscripts are defined as follows: f is fluid; inlet is ι; outlet is o; reference base is r, metal is m, &.E is external or environment. The last term is the fluid velocity head in BTU/lb, if significant.
The variables are:
W = fluid flow rate in mass per time, say lb/hr
T = temperature in degree F
U = inside heat transfer coefficient, say BTU/hr/SF/^{o}F
r = inside pipe radius
c_p = heat capacity, say BTU/lbm/F
ΔX = pipe length, say feet

t = pipe wall thickness in say feet
θ = time unit and to be in consistent units, then it has to be in hours

18. SHELL & TUBE EXCHANGER TEMPERATURE RESPONSE:
The method of derivation follows the above pattern, except there will be 2 simultaneous differential equations. One equation is for the shell side temperature and a second for the tube side temperature. Additionally, the LMTD function will be approximated by a simple average temperature difference. The approximation does not introduce any appreciable error for the majority of industrial application and greatly enhances solution convergence by elimination of the log function.

TUBE SIDE:
Heat in = $[Wc_p]_f(T_i\text{-}T_r)d\theta$ & Heat out = $\{[Wc_p]_f(T_o\text{-}T_r) + UA(LMTD)\}d\theta$

Accumulation = in - out = $\{[Wc_p]_f(T_i\text{-}T_o) - UA(LMTD)\}\ d\theta$

Accumulation = $dT\{(\rho(VOL)cp)_{fluid} + {}^1/_2(Mcp)_{tube}\}$ solving the equality for accumulations:

$dT/d\theta = \{[Wc_p]_f(T_i\text{-}T_o) - UA(LMTD)\}/\ \{(\rho(VOL)cp)_{fluid} + {}^1/_2(Mcp)_{tube}\}$ [½ accounts for 2 fluids]

SHELL SIDE
Heat in = $[Wc_p]_f(T_i\text{-}T_r)d\theta$ & Heat out = $\{[Wc_p]_f(T_o\text{-}T_r) + UA(LMTD)\}d\theta$

Accumulation = in - out = $\{[Wc_p]_f(T_i\text{-}T_o) - UA(LMTD)\}\ d\theta$

Accum. = $dT\{(\rho(VOL)cp)_{fluid}+{}^1/_2[(Mcp)_{tube}+(Mcp)_{shell}]\}$ solving the equality for accumulations:

$dT/d\theta=\{[Wc_p]_f(T_i\text{-}T_o)\text{-}UA(LMTD)\}/\{(\rho(VOL)cp)_{fluid}+{}^1/_2[(Mcp)_{tube}+(Mcp)_{shell}]\}$

For counter current flow the LMTD is approximated as ½(Tot + Tit - Tis - Tos)F. The F is the tube LMTD correction factor, about 0.9 to 1.0. As with the pipe, subtract velocity head, if significant.

Some discussion on the nature of dT/dθ. This is a time forward difference of the bulk average temperature for either shell or tube side. The approximation, used in solutions, is to consider the outlet temperatures as the time forward difference temperature. An exact solution may be obtained by use of partials and an auxiliary equation to obtain the bulk mean temperature.

$dT/d\theta = ({}^{\partial T}/_{\partial x})^{dx}/_{d\theta} + {}^{\partial T}/_{\partial\theta}({}^{d\theta}/_{d\theta}) = (\text{Velocity})({}^{\partial T}/_{\partial x}) + {}^{\partial T}/_{\partial\theta}= (\text{Velocity})({}^{\Delta T}/_{\Delta x}) + {}^{\partial T}/_{\partial\theta}$

Δx = Volume/Area & ΔT = inlet less outlet temperatures, bulk mean temperature is $½(T_{in}+T_{out})$.

19. HEAT EXCHANGE NEAR SONIC VELOCITY WITH HEAT INPUT
A fairly common problem to plant engineers is the analysis of flare systems. In practically all aspects, the flare system is a dynamic analysis. Typically, engineers have sized flare systems on empirical methods to simplify the calculations. It is generally true that empirical methods provide a reliable operation by virtue of over design. Competing for capital funds by over design does not produce optimum results. In the case of plant expansions re-rating an existing system with empirical relations may lead to recommendations to spend capital funds they are not required. The equations for flare system flow analysis tend to fall into either the isothermal camp or huddle around the adiabatic fire. The isothermal camp get just enough heat to keep the gas temperature constant. Whilst the adiabatic gases cannot get any heat, this flow condition is also known as the isenthalpic or constant entropy.

Neither camp is precisely correct. Real gas flow may actually get enough initial heat to increase temperature due to the heat contained in the pipe wall, and then reach a constant temperature point, followed by something approaching adiabatic flow. In small bore pipe, there is more heat flux per flow and also more thermal capacity contained in the pipe walls per unit of flow, with large bore pipe having less of either per unit of flowing capacity. Some analyst have stated that adiabatic conditions are the more conservative, I choose to let the reader decide. The exist velocity will not exceed Mach of one, when flow starts at less than sonic velocity, but just what is the sonic velocity? For adiabatic flow, the maximum downstream Mach number (ratio of flowing velocity to sonic velocity) is one, but for isothermal flow the maximum velocity is sonic velocity over the square root of specific heat ratio (M&S p144, Streeter p361). On this basis, it would appear that isothermal flow capacity is less than the adiabatic flow capacity. If this is true, then it appears that the initial flow conditions are what limit flare capacity, owing to the heat contained in pipe walls. One rule that designers use is to size flare lines on 80% of sonic velocity, this corresponds to a heat capacity ratio of 1.56, something between air and hydrogen. However, for hydrocarbon gases the isothermal factor is 93% of sonic velocity, or 13% more capacity than the 80% rule. Perhaps it would be an improvement to use dynamic analysis to determine flare capacity under the extremes of ambient and inlet flow temperature conditions, while considering the initial heat available from the pipe walls and then finalize the flare capacity, using a conservative pipe roughness, when calculating the friction factor.

For flow approaching sonic velocity, it is best to work either in terms of Mach numbers or mass velocity, G, lbm/sf/sec. Considering 1 dimensional flow it is then possible to define **T** in terms of, $(1/\rho)(G)^2/(2g)$ & **U** = $(p + h\rho J)$. Here h is the enthalpy in BTU/lb of gas and J is 778 ft-lb/BTU, which gives units in pounds per square foot of head. However, it is far more simplistic to include a simultaneous enthalpy balance, as given above and then subtract the velocity head energy from the thermal energy balance plus add the continuity equation, as detailed above. In which case, the correct density for friction loss calculation is the average density. When using mass velocity, then the boundary condition of sonic velocity must be observed, along with any possible heat exchange boundaries or conditions. This method greatly extends the accuracy range of the constant density pressure drop equations to solve variable density flow phenomena, without detracting from the usefulness when density is in fact constant. For example, if adiabatic flow is required, then the adiabatic density relation can be added as an auxiliary condition to solve for outlet temperature by setting U = 0 in the thermal energy balance. If on the other hand, isothermal flow is considered, then a controller can be added to meet the auxiliary condition of inlet temperature must equal outlet temperature. Setting up a series of pipe lengths then adding additional segments of decreasing length to keep total length constant until the last added length does not alter overall pressure drop is a simple method to check solution validity.

The attached graphs provides a comparison of this method, to that of Example 10-4 of Smith & Van Ness, note that S&VN did not include the effects of metal thermal capacity, as their interest was in steady state, not dynamic solutions. The solution by S&VN with 170 psi inlet pressure was 140 psid while this solution arrived at 130 psid. The two answers were close approximations so no further runs were made to determine the effect of decreasing the segment size.

The temperature rise from a heat exchanger must consider the thermal mass of the exchanger. The typical models (Ramez, Dynamic Process Simulation, Example 5.3) neglect the thermal mass of the exchanger shell and tubes to simplify the calculation procedures. With the computing improvements available now, these improvements should be incorporated into the dynamic calculations.

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